

# NATURAL GAS & ELECTRIC: THE INTEGRATED RESOURCES OF ENERGY

Gokturk Poyrazoglu<sup>1</sup>, HyungSeon Oh<sup>2</sup>

<sup>1</sup>Electrical & Electronics Engineering Department, Faculty of Engineering, Ozyegin University, Istanbul, Turkey

<sup>2</sup>Electrical Engineering Department, School of Engineering and Applied Sciences, SUNY-Buffalo, Buffalo, NY, USA

Corresponding author: Gokturk Poyrazoglu, e-mail: [gokturk.poyrazoglu@ozyegin.edu.tr](mailto:gokturk.poyrazoglu@ozyegin.edu.tr)

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REFERENCE NO	ABSTRACT
MANG-03	The use of natural gas as a resource in generating electricity is increasing substantially in Europe, reducing the fuel mix share of other fossil based resources. The reason of this change is supported by the environmental causes and the economical aspect. To sustain the growth and to reach the real potential, the impact of natural gas networks on power grid requires simulation based research. This study addresses the reliability aspect of Integrated Energy Resources by adopting the multi-objective optimization approach. Mathematical programming of both systems are developed and two scenarios covering the challenges against natural disasters are discussed in detail.

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## 1. INTRODUCTION

Clean and safe energy is more important than ever in today's world for development, social welfare, and humanity. Energy industry is growing financially. The renewable energy sector employed over one million people in Europe and created a turnover of around €140 billion in 2014. Each year 500,000 new jobs are expected to be filled in energy industry. While it holds a substantial growth in business, new technologies inserted to the traditional operation scheme create challenges to maintain high efficiency and affordability. To sustain the growth and to reach the real potential, individual energy networks such as natural gas and electricity must be operated, traded, and planned in the framework of Integrated Energy Networks (IEN). This study focuses on the large-scale operation and planning of IEN under the optimisation umbrella to address the challenges on the reliability aspect of gas and electric systems against natural disasters. The reliability in gas and electric is not well studied in the literature [1-3] in comparison to the economic aspects of co-optimization [4-16].

The use of natural gas in fuel mix of electricity generation has been continually increasing since 1990s. This is due to the decreasing cost of natural gas as well as its relatively low impact on air pollution and carbon dioxide emission, the main contributors to the climate change problems.

However, the demand of natural gas is not only related to electricity generation, it is mainly driven by heating related demand. At the time of high demand on natural gas, such as extreme weather conditions, the supply of natural gas is either enough but expensive or not enough to satisfy the needs for both buildings and power plants. This raises a reliability problem in both gas and power systems; and if not studied and prepared correctly would cause service interruptions and even blackouts. The infrastructure and the connections are already in the field; however, the efficient and optimised integration of systems can offer new opportunities to create resilient and secure energy network.

Traditionally electric and gas are treated as a commodity and traded individually. Their systems are operated independently. This study focuses on the future of energy systems and it lies on the idea of integrating resources of energy to optimise operation for the goal of maximizing efficiency, affordability, and social welfare.

Multi-objective optimization techniques are utilized in this study to show the impact of different scenarios on system variables such as price, unserved energy, unserved gas, fuel mix, and the reliability indices. The study also includes the stochastic nature of variable energy resources (VER) such as wind and solar. Stochastic approach on the problem has been studied and analyzed in literature for

different objectives and constraints [17-23]. The uncertainty and variability of VER units at the time of their future production forecast can be addressed by utilizing a stochastic approach. The forecasts of VERs have a direct impact on the net load expectation, which is used to determine the failure probability of the system.

## 2. MULTI-OBJECTIVE OPTIMIZATION

The nature of multi-objective optimization is to consider two or more dependent or independent objectives simultaneously to understand the possible correlation between them. A pareto efficient solution of a problem is a state of allocation of resources in which it is impossible to make any one individual objective better off without making at least one individual worse off. The pareto front; on the other hand, as is used in the multi-objective optimization is a term that explains possible outcomes that are all pareto efficient. By only focusing on the set of choices that are pareto efficient, the decision maker can make tradeoffs within this set. Since all optimization objectives used in this study are in the same direction, the pareto front reveals the best possible outcomes with the given conditions. To incorporate a multi-objective optimization

framework, a weighting approach on each system's production cost is adopted. While the total weight is unity for all cases, the individual weights changing by 0.1 intervals to create total of 9 cases.

The expected value of this study is to show the energy industry and power communities that early actions may prevent some, if not all, undesired outcomes of considering both systems independently.

## 3. MATHEMATICAL MODELS

The co-optimization model utilizes the multi-objective optimization approach in order to prioritize the importance of each objective based on the operational requirements. For the sake of clarity, electrical and mechanical models are given separately later in this chapter. The integration of both models are performed via weighting approach and various weight combinations are tested and presented. Power systems is modeled to address the unit commitment and optimal power flow in a single optimization problem. The objective function in (10) minimizes the total cost of the system. The production, no-load and startup costs of each generator, and the cost of unserved load are considered. The multiplication of the former and latter term are

### SECURITY CONSTRAINED ECONOMIC POWER DISPATCH PROBLEM

$$\min_{P_k, P_g, \theta, s, u, s^{USE}} \sum_{g,t} c_g(P_{g,t}^G) + l_g(u_{g,t}) + r_g(s_{g,t}) + \sum_{n,t} z_n(s_{n,t}^{USE}) \quad (10)$$

Subject to

$$P_{k,t} = B_k(\theta_i - \theta_j) \quad (1)$$

$$\sum_{g \in n} P_{g,t}^G - D_{n,t} + s_{n,t}^{USE} = \sum_{k \in (n,*)} P_{k,t} - \sum_{k \in (*,n)} P_{k,t} \quad (2)$$

$$s_{n,t}^{USE} \geq 0 \quad (3)$$

$$-R_{g,t}^{Down} \leq P_{g,t}^G - P_{g,(t-1)}^G \leq R_{g,t}^{Up} \quad (4)$$

$$P_g^{Min} * u_{g,t} \leq P_{g,t}^G \leq P_g^{Max} * u_{g,t} \quad (5)$$

$$-P_k^{Max} \leq P_{k,t} \leq P_k^{Max} \quad (6)$$

$$s_{g,t} = u_{g,t} - u_{g,(t-1)} \quad (7)$$

$$\sum_{\tau=t-\rho_g+1}^t s_{g,\tau} \leq u_{g,t} \quad (8)$$

$$\sum_{\tau=t-\mu_g+1}^t s_{g,\tau} \leq 1 - u_{g,\gamma} \quad (9)$$

where  $c_g(P_{g,t}^G) = (b_g \times P_{g,t}^G + a_g) \times \beta$ ,  $l_g(u_{g,t}) = u_{g,t} \times NL_g$ ,  $r_g(s_{g,t}) = s_{g,t} \times SU_g$ ,  $z_n(s_{n,t}^{USE}) = s_{n,t}^{USE} \times VoLL \times \beta$ ,  $\gamma = t - \mu_g$ ,  $\beta$ : baseMVA

by baseMVA  $\beta$  is to use the same unit in the function.

(1) is a function of voltage angle difference between the two connected buses and the susceptance of the line.

(2) creates the balance between the incoming and outgoing flow from a node with an introduction of a non-negative slack variable in (3) that addresses the unserved energy at the node.

(4) ramping capability of each generator.

(5) min and max production capacities of each generator. The binary commitment variable ( $u$ ) enforces the zero generation during de-commitment.

(6) max transfer capability of each transmission line.

(7) the unit commitment logic of the system,

(8) the minimum time to restart a generator,

(9) the minimum time to de-commit a generator.

Secondly, gas-system model creates an economic supply and demand balance while considering the limitations of infrastructure.

The objective function in (23) minimizes the total production cost of the system. The first two terms consider the unserved gas demand from heating and electrical facilities respectively. The economic dispatch of gas

wells is addressed in the third term. Gas storage operations are considered by the last two terms.

(11) creates a nodal gas demand-supply balance.

(12) limits the nodal min. and max. demand considering the total sum of heating and electrical based gas-demands and unserved pieces.

(13) limits the gas production at each well.

(14) limits the flow capability at each pipeline.

The gas consumption of a gas-fired unit depends on hourly power dispatch. So it is modeled in (15) as a quadratic function of the unit's power production; hence the resultant gas demand is represented as the extracted gas in (11). The linearized Weymouth equation in (16) represents the gas flow rate across a pipeline; and it is determined by the difference of the pressures between the terminating gas nodes. The pressure and the unserved energy variables must be always non-negative and to the pressure at one node is set to 1 per unit psia as a reference.

(19) limits the amount of gas in the storage.

(20) secures the state of storage.

(21) limits the unserved gas demand by electrical facilities.

## ECONOMIC GAS DISPATCH PROBLEM

$$\min_{v, l^{elect}, f, s, g, p, e} \sum_t \left( \sum_n s_{n,t}^{heat} * VOLD^{heat} + \sum_g s_{g,t}^{elect} * VOLD^{elect} + \sum_s v_{s,t} * C_s + \sum_u g_{u,t}^{net} * SP_u \right) \quad (23)$$

Subject to

$$\sum_{s \in n} v_{s,t} + \sum_{k \in (*,n)} f_{k,t} = h(.) + \sum_{u \in n} g_{u,t}^{net} \sum_{k \in (n,*)} f_{k,t} \quad (11)$$

$$\underline{l}_n \leq h(.) \leq \bar{l}_n \quad (12)$$

$$\underline{v}_s \leq v_{s,t} \leq \bar{v}_s \quad (13)$$

$$\underline{f}_k \leq f_{k,t} \leq \bar{f}_k \quad (14)$$

$$l_{g,t}^{elect} = a_g + b_g P_{g,t}^G + c_g (P_{g,t}^G)^2 \quad (15)$$

$$f_{k,t} * K_k = \varphi_i * p_{i,t} - \varphi_j * p_{j,t} \quad (16)$$

$$p_{ref,t} = 1 \quad (17)$$

$$p_{n,t} \geq 0 \quad (18)$$

$$\underline{e}_u \leq e_{u,t}^{prev} \leq \bar{e}_u \quad (19)$$

$$e_{u,t}^{prev} - e_{u,(t-1)}^{prev} = g_{u,t}^{net} \quad (20)$$

$$0 \leq s_{g,t}^{elect} \leq l_{g,t}^{elect} - a_g \quad (21)$$

$$0 \leq s_{n,t}^{heat} \leq l_{n,t}^{heat} \quad (22)$$

where  $h(.) = l_{n,t}^{heat} - s_{n,t}^{heat} + \sum_{g \in n} l_{g,t}^{elect} - \sum_{g \in n} s_{g,t}^{elect}$ ,  $g_{u,t}^{net} = g_{u,t}^{stor} - g_{u,t}^{inject}$

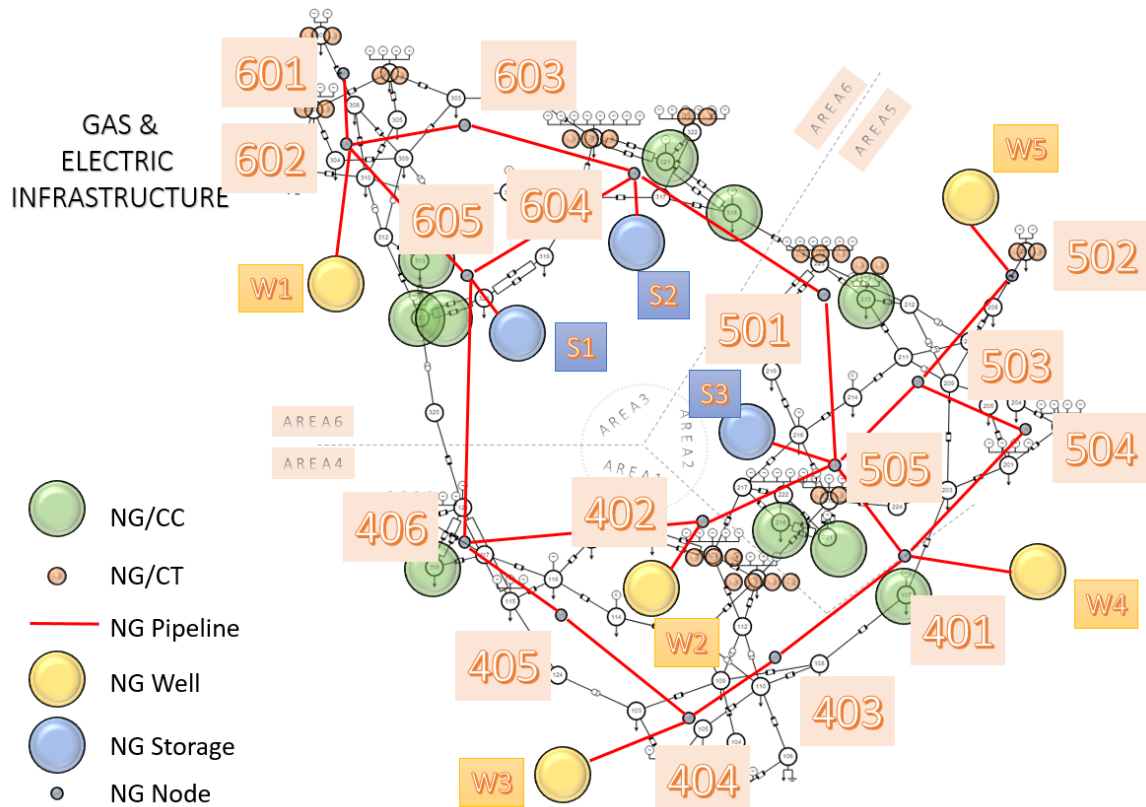


Figure 1. The gas infrastructure parameters utilized in this study

(22) limits the unserved gas demand by heating

#### 4. CASE STUDIES

An IEEE test system with justifiable updates is used to reflect the realistic gas and electric environment, yet the models presented in this research may be applied to the large-scale power systems. The base case is used to create a scenario in which the system faces a power service interruption due to the non-existence of co-optimization in gas and electric systems. The study focuses on the actions taken ahead of time to prevent interruption being occurred. Possible actions for example include (i) advanced commitment of thermal generators other than gas-fired plants, (ii) early activation of demand response units, (iii) increasing the ramp flexibility of the system by introducing load following reserves, (iv) advanced storage of gas at the locations next to high load pockets.

Although the possibility of creating a case study on gas-electric co-optimization studies is endless, this paper focuses on two cases that may address wide range of questions related to system optimizations. This study compares

the simulation results gathered by utilizing the first timeline illustrated in Figure 2 and by utilizing the co-optimized timeline shown in Figure 3.

The first one is related to the demand variability and uncertainty between the day-ahead forecast and real time actuals. The demand is assumed to be under-estimated at the day-ahead forecast; so, the actual need is greater in real time due to an environmental impact, for example, a cold storm leading to run more heating units than usual that increases both gas and electric demand.

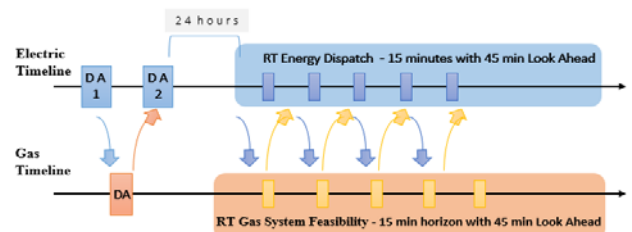


Figure 2. Timeline for operation of gas and electric systems individually and information transfer

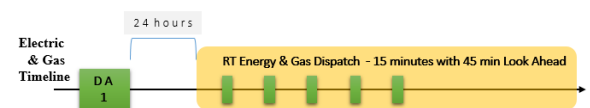


Figure 3. Timeline for co-optimized gas and electric systems

The second case study is related to a sudden and unexpected change on the gas infrastructure that may be thought as a forced outage of a pipeline, or as an accident that prevents the gas transfer between two points.

#### 4.A. Demand variability and uncertainty

To address the variability and uncertainty in the demand among different scheduling processes e.g. Day-ahead and Real Time, a scenario of having a severe temperature drop which leads high use of heating devices is considered in this case study. Although those natural events may have been forecasted in advance by the meteorologists, the consumer experience shows different reaction to each event that leads a risky operating condition for both systems. In this scenario, real time demand is increased by up to 3% for gas, and 5% for electric.

The unit commitment is only allowed on DA so no RT commitment in the systems. Due to the increase on both heating related demands, it is expected to observe (i) gas curtailment to gas-fired units, (ii) less power from gas-fired units, (iii) more power from other units, and (iv) possible loss load in power systems due to lack of head room capacity. These changes on the system conditions would make the systems' production cost volatile especially in power systems.

All expectations actually be observed in the simulation results. To recall the structure of the simulations, the weight that is shown as "w1" represents low priority to gas production cost and high priority on electric production cost, however, the "w9" represents high priority of gas, and low priority of electric.

As the priority of electric cost is getting lower, the variability and as well as the distribution of real time electric cost is increasing. The gas system; however, doesn't observe a significant volatility to the same changes. While the priority of gas is increasing, the median production cost slightly decreases.

Another observation can be illustrated in a multi-objective approach. A scatter chart for that purpose is plotted in Figure 4, which has total real time production cost of gas system on the y-axis and the total real time

production cost of electrical system on the x-axis. This illustration reveals the pareto frontier results of co-optimization. The pareto frontier is the set of Pareto efficient allocations, which it is impossible to reallocate so as to make any individual objective better off without worsen off the other ones. The use of lowest priority on gas and the highest priority on electric (w1) is observed to be the best option to minimize the

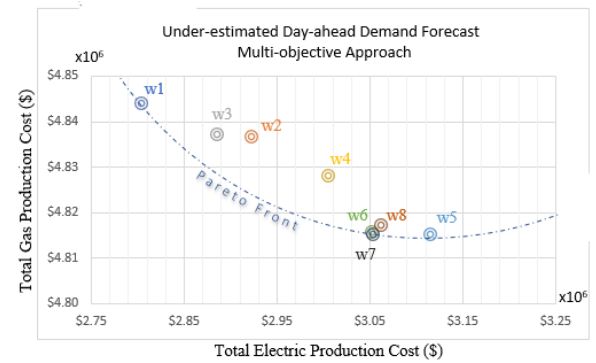


Figure 4. Pareto front chart - the case of under-estimated day-ahead demand forecast

overall production cost. This observation is consistent with the increasing volatility seen in the power systems.

#### 4.B. Sudden and unexpected change

Unlike the regular scheduled maintenance on infrastructure, the forced outage of an asset may cause severe consequences to the system reliability. Gas and electric systems are always vulnerable to such sudden events; therefore, in this scenario two separate pipelines are assumed to be offline for a limited time starting in different time of day. These pipelines are selected to be the most utilized in terms of capacity factor. One of them is connecting Gas Node 402 to 505 as shown in Figure 1. This pipeline is used in its maximum capacity almost every hour of day due to the significant gas demand from gas-fired units around. The second pipeline is connecting Gas Node 503 to 504. This pipeline serves high heating related gas demand at Node 504. The first pipeline is assumed to be out between 4am and 8am and the second is out between 1pm and 6pm.

This scenario doesn't have an impact on the DA due to the lack of knowledge in advance;

however, the impact is visible in the RT. The following metrics are evaluated: (a) unserved electric energy, (b) cost volatility, and (c) total production cost.

The unserved energy is the imbalance of nodal power injection and ejection in the system. Although not to serve the demand is not a regular application, it is a mathematical expression of stating the infeasibility of serving the load under the current system conditions, or the feasibility of shedding load with a high penalty in a tradeoff to minimize the total production cost.

The results show that the assumed gas infrastructure is flexible to accommodate the loss of a pipeline by changing the dispatch of gas wells, so no interruption is observed in satisfying the heating related gas demand; however, deliverability to the gas-fired units is interrupted due to the reduction on gas transfer capacity. This gas curtailment evidently reduces the power output of gas-fired units in RT. The chain of consequences finally arrives the electric system and causes a problem on meeting the nodal demand in RT. The highest two unserved energy measures are observed when the priority of gas is the highest and the lowest, w9 and w1 respectively. The relatively low loss load is observed when the weights are close to each other such as in w5, w6, w7.

The unserved energy measures are consistent with variance on the production cost results. The pareto front in Figure 5 shows that the high priority of electric awards the financial recovery of the electric system rather than the physical recovery of the gas system to increase gas transfer to power units causing high unserved energy and evidently high production cost to the electrical system.

On the other hand, opposite consequence is observed when the priority of gas is becoming dominant on the objective function, e.g. w7, w8. Dominant financial recovery on gas cost neglects the physical recovery of electric system that evidently increases the unserved energy and the production cost. The optimal weighting is observed when the weight of w6 is used, which is not the equal weight

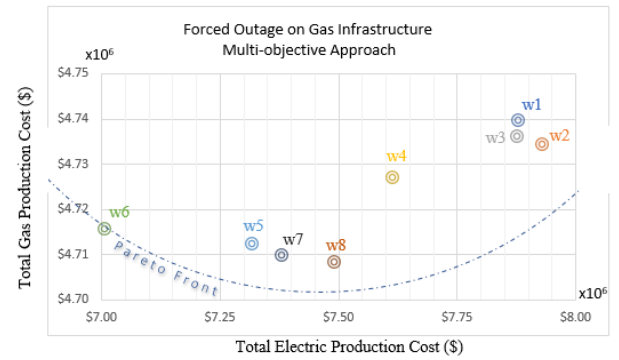


Figure 5. Pareto front chart - the case of forced outage on gas infrastructure

condition but provides an insight to the system conditions.

The volatility study also supports the previous findings. Although the contingency is in the gas infrastructure, gas system is more stable to respond to the failure in comparison to the electric system. As a result, the reduction on the gas deliverability to the power units increases the volatility of RT electric production cost. Consistently with the other findings, the closer the weights of both systems are the volatility increases with a tradeoff of having lower production cost in the electrical system; however, the gas cost shows negligible variance even at the time of forced outages on the pipelines.

## 5. CONCLUSIONS

Due to a continuous increase in the consumption of natural gas to generate electricity, both systems are connected more than ever in the history. However, the operational infrastructure of both systems were not designed to have such a bound between them. The mathematical models of each system in terms of their daily operation scheme are modelled in this study and co-optimized operations are simulated to understand the impact of demand variability and uncertainty as well as sudden and unexpected change in the system parameters. The results show that the gas infrastructure is more mature and have flexibility against the price volatility; however, the electric grid is now under more risk although the sudden change only occurs in the gas network. The multi-objective approach for the operations of

both systems also supports the findings above and reveals that the The use of lowest priority on gas and the highest priority on electric is observed to be the best option to minimize the overall production cost.

## Nomenclature

### Indexes

$k$	transmission lines
$i$	gas nodes
$n$	the end node of a transmission line
$t$	time interval
$g$	generator
$s$	gas wells
$u$	gas storage

### Parameters

$B_k$	susceptance
$D_{n,t}$	demand (load)
$R_{g,t}^{Down}$	ramp down limit
$R_{g,t}^{Up}$	ramp up limit
$P_g^{Max}$	maximum generation limit
$P_g^{Min}$	minimum generation limit
$P_k^{Max}$	maximum power transmission limit
$\rho_g$	minimum up time
$\mu_g$	minimum down time
VoLL	value of loss load
$\beta$	base MVA
$NL_g$	no load cost
$a_g$	gas demand coefficient
$b_g$	gas demand coefficient
$SU_g$	start-up cost
$l_{n,t}^{heat}$	heating related gas demand
$\underline{l}_n$	minimum nodal gas demand
$\bar{l}_n$	maximum nodal gas demand
$\underline{v}_s$	minimum gas well delivery
$\bar{v}_s$	maximum gas well delivery
$\underline{f}_k$	minimum gas flow limit
$\bar{f}_k$	maximum gas flow limit
$K_k$	pressure coefficient
$\varphi_i$	nodal pressure flow coefficient
$\underline{e}_u$	minimum gas storage value
$\bar{e}_u$	maximum gas storage value
$VOLD^{heat}$	value of loss gas demand (heating)
$VOLD^{elect}$	value of loss gas demand (electric)
$C_s$	gas delivery cost
$SP_u$	gas storage delivery cost

### Continuous Variables

$\theta_i$	voltage angle
$P_{k,t}$	power flow
$P_{g,t}^G$	power generation
$S_{n,t}^{UsE}$	unserved energy
$v_{s,t}$	gas production
$f_{k,t}$	gas flow
$g_{u,t}^{net}$	net gas storage injection
$S_{n,t}^{heat}$	unserved heating related gas demand
$l_{g,t}^{elect}$	electric related gas demand
$p_{n,t}$	nodal pressure
$e_{u,t}^{gas}$	current gas storage state

### Binary Variables

$u_{g,t}$	unit commitment
$S_{g,t}$	start-up/shutdown decision

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